

Futures Options

Chapter 16

Mechanics of Call Futures Options

When a call futures option is exercised the holder acquires

1. A long position in the futures
2. A cash amount equal to the excess of
the futures price over the strike price

Mechanics of Put Futures Option

When a put futures option is exercised the holder acquires

1. A short position in the futures
2. A cash amount equal to the excess of
the strike price over the futures price

The Payoffs

If the futures position is closed out immediately:

$$\text{Payoff from call} = F_0 - K$$

$$\text{Payoff from put} = K - F_0$$

where F_0 is futures price at time of exercise

Potential Advantages of Futures Options over Spot Options

- Futures contracts may be easier to trade than underlying asset
- Exercise of option does not lead to delivery of underlying asset
- Futures options and futures usually trade side by side at an exchange
- Futures options may entail lower transactions costs

Put-Call Parity for Futures Options

(Equation 16.1, page 345)

Consider the following two portfolios:

1. European call plus Ke^{-rT} of cash
2. European put plus long futures plus cash equal to F_0e^{-rT}

They must be worth the same at time T so that

$$c + Ke^{-rT} = p + F_0 e^{-rT}$$

Other Relations

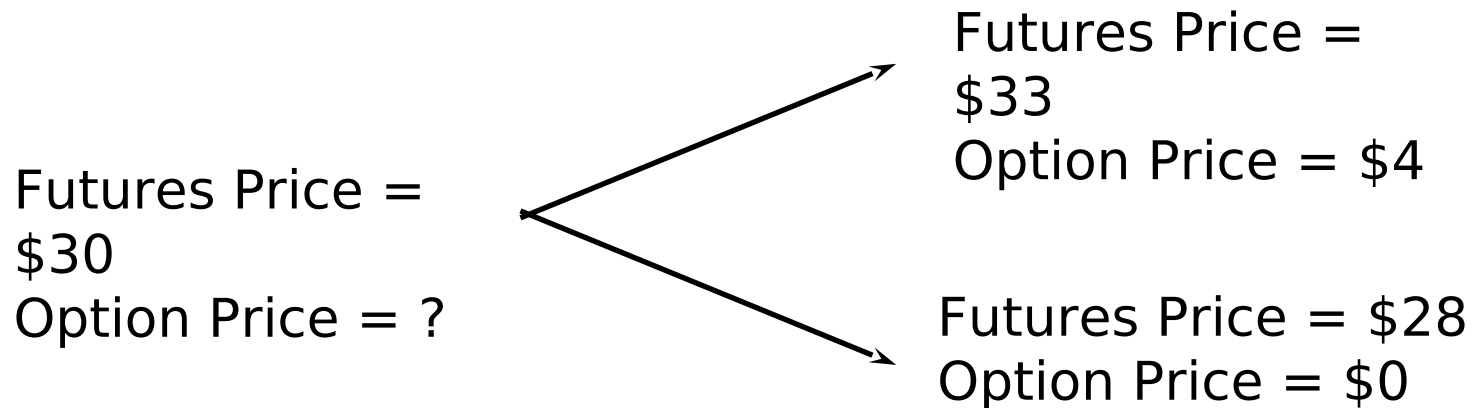
$$F_0 e^{-rT} - K < C - P < F_0 - Ke^{-rT}$$

$$c > (F_0 - K)e^{-rT}$$

$$p > (F_0 - K)e^{-rT}$$

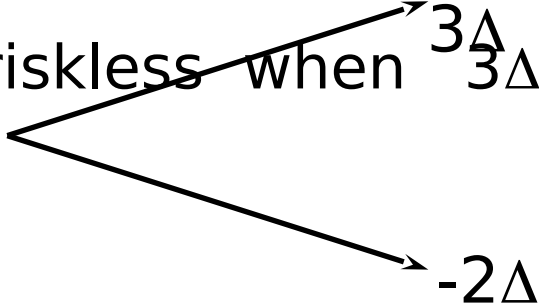
Binomial Tree Example

A 1-month call option on futures has a strike price of 29.



Setting Up a Riskless Portfolio

- Consider the Portfolio: long Δ futures short 1 call option

- Portfolio is riskless when $3\Delta - 4 = -2\Delta$ or $\Delta = 0.8$
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Valuing the Portfolio

(Risk-Free Rate is 6%)

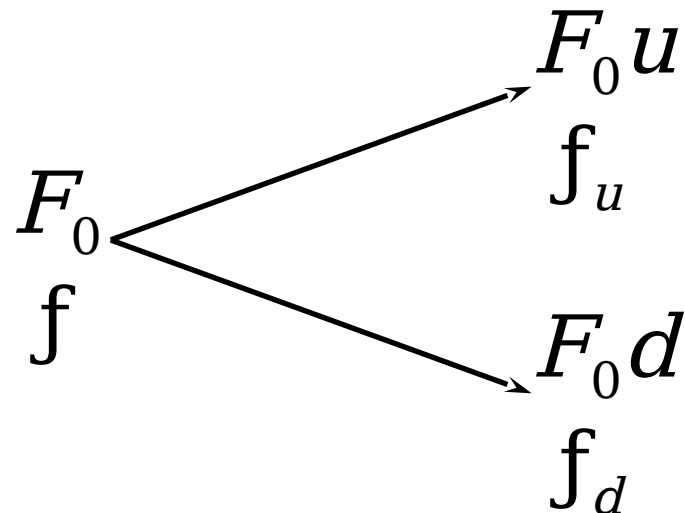
- The riskless portfolio is:
 long 0.8 futures
 short 1 call option
- The value of the portfolio in 1 month is
 -1.6
- The value of the portfolio today is
 $-1.6e^{-0.06/12} = -1.592$

Valuing the Option

- The portfolio that is
 long 0.8 futures
 short 1 option
is worth -1.592
- The value of the futures is zero
- The value of the option must
therefore be 1.592

Generalization of Binomial Tree Example (Figure 16.2, page 348)

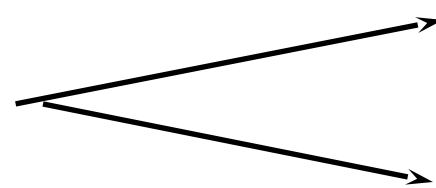
- A derivative lasts for time T and is dependent on a futures price



Generalization

(continued)

- Consider the portfolio that is long Δ futures and short 1 derivative


$$\begin{aligned} & F_0 u \Delta - F_0 \Delta - f_u \\ & F_0 d \Delta - F_0 \Delta - f_d \end{aligned}$$

- The portfolio is riskless when

$$\Delta = \frac{f_u - f_d}{F_0 u - F_0 d}$$

Generalization

(continued)

- Value of the portfolio at time T is $F_0 u \Delta - F_0 \Delta - f_u$
- Value of portfolio *today* is $-f$
- Hence

$$f = - [F_0 u \Delta - F_0 \Delta - f_u] e^{-rT}$$

Generalization

(continued)

- Substituting for Δ we obtain

$$f = [p f_u + (1 - p) f_d] e^{-rT}$$

where

$$p = \frac{1 - d}{u - d}$$

Growth Rates For Futures Prices

- A futures contract requires no initial investment
- In a risk-neutral world the expected return should be zero
- The expected growth rate of the futures price is therefore zero
- The futures price can therefore be treated like a stock paying a dividend yield of r

Valuing European Futures Options

- We can use the formula for an option on a stock paying a dividend yield

Set $S_0 =$ current futures price
(F_0)

Set $q =$ domestic risk-free rate (r)

- Setting $q = r$ ensures that the expected growth of F in a risk-neutral world is zero

Black's Model

(Equations 16.9 and 16.10, page 350)

- Black's model provides formulas for European options on futures

$$c = e^{-rT} [F_0 N(d_1) - K N(d_2)]$$

$$p = e^{-rT} [K N(-d_2) - F_0 N(-d_1)]$$

$$\text{where } d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln(F_0 / K) - \sigma^2 T / 2}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T}$$

How Black's Model is Used in Practice

- European futures options and spot options are equivalent when futures contract matures at the same time as the option
- This enables Black's model to be used to value a European option on the spot price of an asset

Using Black's Model Instead of Black-Scholes (Example 16.7, page 351)

- Consider a 6-month European call option on spot gold
- 6-month futures price is 620, 6-month risk-free rate is 5%, strike price is 600, and volatility of futures price is 20%
- Value of option is given by Black's model with $F_0=620$, $K=600$, $r=0.05$, $T=0.5$, and $\sigma=0.2$
- It is 44.19

Futures Style Options (page 352-53)

- A futures-style option is a futures contract on the option payoff
- Some exchanges trade these in preference to regular futures options
- A call futures-style option has value

$$F_0 N(d_1) - KN(d_2)$$

- A put futures style option has value

$$KN(-d_2) - F_0 N(-d_1)$$

Futures Option Prices vs Spot Option Prices

- If futures prices are higher than spot prices (normal market), an American call on futures is worth more than a similar American call on spot. An American put on futures is worth less than a similar American put on spot
- When futures prices are lower than spot prices (inverted market) the reverse is true

Put-Call Parity Results

Indices

$$c + K e^{-rT} = p + S_0 e^{-qT}$$

Foreigntaxchange

$$c + K e^{-rT} = p + S_0 e^{-r_f T}$$

Futures:

$$c + K e^{-rT} = p + F_0 e^{-rT}$$

Summary of Key Results from Chapters 15 and 16

- We can treat stock indices, currencies, and futures like a stock paying a dividend yield of q
 - For stock indices, q = average dividend yield on the index over the option life
 - For currencies, $q = r_f$
 - For futures, $q = r$